

# Design and Comparative Study of Digital FIR Filters using Improved Particle Swarm Optimization

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**Abstract** – In order to reduce the effect of Inter Symbol Interference (ISI) in a digital communication system, different types of transmitting pulse shaping filters have been extensively used in practice. We need to design these filters with some constraints imposed by requirements of the communication system in which we are going to use them. The use of optimization techniques have been proved to be quite useful towards the design of those digital filters with certain specifications. This paper proposes a framework for designing; Low Pass, High Pass, Band Pass and Band-Stop filters, using Improved Particle Swarm Optimization (IPSO). IPSO is an improved PSO that proposes a new definition for the velocity vector and swarm updating and hence the solution quality is improved. In the design process, the filter length, pass band and stop band frequencies, feasible pass band and stop band ripple sizes are specified. The performance of the proposed filter has been examined by recording the variation of the resulting frequency response with number of iteration and population size.

**Keywords**– Inter Symbol Interference, Improved Particle Swarm Optimization, Low Pass, High Pass, Band Pass and Band-Stop Filters.

## I. INTRODUCTION

A filter is a frequency selective circuit that allows a certain frequency to pass while attenuating the others. Filters could be analog or digital. Analog filters use electronic components such as resistor, capacitor, transistor etc. to perform the filtering operations. These are mostly used in communication for noise reduction, video/audio signal enhancement etc.

Digital filters are used in wide variety of applications from signal processing, aerospace, control systems, defence equipment, telecommunications, system for audio and video processing to systems for medical applications to name just a few. Basically filter refers to a frequency selective device which extracts the useful portion of input signal lying within its operating frequency range and could be contaminated with random noise due to unavoidable circumstances. Analog filters are implemented with discrete components but the digital filters perform mathematical operations on a sampled, discrete time signal to reduce or enhance the desired features of the applied signal [1].

Digital filters are superior to their analog counterpart due to its wide range of applications and better performance. The advantages of digital filters over analog filters are small physical size, high accuracy and reliability. Digital filtering is one of the most powerful tools of Digital Signal Processing. Digital filters are capable of performance specifications such as ability to achieve multi-rate operation and exact linear phase that would, at best, be extremely difficult, if not impossible, to achieve with an analog implementation. In addition, digital filter characteristics are easy to change under software control. Digital filters are widely used in the fields of automatic control, telecommunications, speech processing and many more. Digital filters are of two types; finite impulse response (FIR) and infinite impulse response (IIR) filter which may also take on the names non-recursive and recursive, respectively.

The main advantage of the FIR filter structure is that it can achieve exactly linear-phase frequency responses. That is why almost all design methods described in the literature deal with filters with this property. Since the phase response of linear-phase filters is known, the design procedures are reduced to real-valued approximation problems, where the coefficients have to be optimized with respect to the magnitude response only. These coefficients are to be optimized using evolutionary algorithms such as Genetic Algorithm (GA) or Particle Swarm Optimization etc. The principle objective of this paper is to propose a framework for designing; Low Pass, High Pass, Band Pass and Band-Stop filters, using Improved Particle Swarm Optimization. An iterative method, Improved Particle Swarm Optimization (IPSO) is introduced to find the optimal solution of filter design problem.

## II. DESIGNING OF FILTERS

A digital FIR filter is characterized by:

$$H(z) = \sum_{n=0}^N h(n)z^{-n}, \quad n = 0, 1, \dots, N \quad (1)$$

Where  $N$  is the order of the filter which has  $(N + 1)$  number of filter's impulse response coefficients,  $h(n)$ . The values of  $h(n)$  will determine the type of the filter, e.g., low pass, high pass, band pass etc. The values of  $h(n)$  are to be determined in the design process and  $N$  represents the order of the polynomial function.

Because the  $h(n)$  coefficients are symmetrical, the dimension of the problem is halved. Thus,  $(N/2 + 1)$  number of  $h(n)$  coefficients are actually optimized, which are finally concatenated to find the required  $(N+1)$  number of filter coefficients. An ideal filter has a magnitude of one in the pass band and a magnitude of zero in the stop band. Error fitness function is formed by the errors between the frequency responses of the ideal filter and the designed approximate filter. In each iteration of the optimization algorithm, error fitness values of particle vectors are calculated and used for updating the particle vectors with new coefficients  $h(n)$ . The final particle vector obtained after a certain number of iterations or after the error fitness is below a certain limit is considered to be the optimal result, yielding an optimal filter. Various filter parameters which are responsible for the optimal filter design are stop band and pass band normalized frequencies ( $\omega_s, \omega_p$ ), pass band and stop band ripples ( $\delta_p$  and  $\delta_s$ ), stop band attenuation and transition width. These parameters are decided by the filter coefficients. Several scholars have investigated and developed algorithms in which  $N$ ,  $\delta_p$ , and  $\delta_s$  are fixed while the remaining parameters are optimized.

In this research work, PSO and Improved PSO (IPSO) are individually applied to obtain the actual designed filter response as close as possible to the ideal response. Now for (1), the particle i.e. the coefficient vector  $\{h_0, h_1, \dots, h_N\}$ , which is optimized, is represented in  $(N/2 + 1)$  dimension instead of  $(N + 1)$  dimension. The frequency response of the FIR digital filter can be calculated as,

$$H(e^{j\omega_k}) = \sum_{n=0}^N h(n)e^{-j\omega_k n} \quad (2)$$

Where,  $\omega_k = \frac{2\pi}{N}$ ;  $H(e^{j\omega_k})$  is the Fourier transform complex vector. This is the FIR filter's frequency response. The frequency is sampled in  $[0, \pi]$  with  $N$  points. Different kinds of error fitness functions have been used in different literatures. An error function given by (3) is the approximate error used for filter design.

$$E(\omega) = G(\omega)[H_d(e^{j\omega}) - H_i(e^{j\omega})] \quad (3)$$

Where  $H_d(e^{j\omega})$  is the frequency response of the designed approximate filter;  $H_i(e^{j\omega})$  is the frequency response of the ideal filter;  $G(\omega)$  is the weighting function used to provide different weights for the approximate errors in different frequency bands.

For ideal LP filter,  $H_i(e^{j\omega})$  is given as:

$$\begin{aligned} H_i(e^{j\omega}) &= 1 \text{ for } 0 < \omega < \omega_c; \\ &= 0 \text{ Otherwise} \end{aligned} \quad (4)$$

For ideal HP filter,  $H_i(e^{j\omega})$  is given as:

$$\begin{aligned} H_i(e^{j\omega}) &= 0 \text{ for } 0 < \omega < \omega_c; \\ &= 1 \text{ Otherwise} \end{aligned} \quad (5)$$

Where  $\omega_c$  is the cut-off frequency. The major drawback of this approach is that the ratio of  $\delta_p/\delta_s$  is fixed. To improve the flexibility in the error function to be minimized, so that the desired level of  $\delta_p$  and  $\delta_s$  may be specified, the error function given in (6) has been considered as fitness function. The error fitness to be minimized using the evolutionary algorithms, is defined as:

$$J_1 = \max_{\omega \leq \omega_p} (|E(\omega)| - \delta_p) + \max_{\omega \geq \omega_s} (|E(\omega)| - \delta_s) \quad (6)$$

Where  $\delta_p$  and  $\delta_s$  are the ripples in the pass band and stop band, respectively, and  $\omega_p$  and  $\omega_s$  are pass band and stop band normalized cut-off frequencies, respectively. Since the coefficients of the linear phase positive symmetric even order filter are matched, the dimension of the problem is halved. This greatly reduces the computational burdens of the algorithms.

In this research work, a novel error fitness function given by (7) has been adopted in order to achieve higher stop band attenuation and to have better control on the transition width. By using (7), it is found that the proposed filter design approach results in considerable improvement over other optimization techniques.

$$\begin{aligned} J_2 &= \sum abs[abs\{H_d(\omega) - 1\} - \delta_p] \\ &\quad + \sum [abs\{H_d(\omega) - \delta_s\}] \end{aligned} \quad (7)$$

For the first term of equation (7),  $\omega \in$  pass band including a portion of the transition band and for the second term of equation (7),  $\omega \in$  stop band including the rest portion of the transition band. The portions of the transition band chosen depend on pass band edge and stop band edge frequencies.

The error fitness function given in equation (7) represents the generalized fitness function to be minimized using the evolutionary algorithms PSO, and the proposed IPSO individually. Each algorithm tries to minimize this error fitness  $J_2$  and thus optimizes the filter performance. Unlike other error fitness functions which consider only the maximum errors,  $J_2$  involves summation of all absolute errors for the whole frequency band, and hence, minimization of  $J_2$  yields much higher stop band attenuation and lesser stop band ripples. Transition width is also kept reduced. Since the coefficients of the linear phase filter are matched, the dimension of the problem is halved. This largely reduces the computational complexities of the algorithms, applied to the optimal design of linear phase positive even symmetrical FIR filters.

### III. PARTICLE SWARM OPTIMIZATION (PSO)

In PSO, a neighbourhood is defined for each individual particle as the subset of particles which it is able to communicate with. The first PSO model used a Euclidian neighbourhood for particle communication, measuring the actual distance between particles to determine which were close enough to be in communication. This was done in imitation of the behaviour of bird flocks, similar to biological models where individual birds are only able to communicate with other individuals in the immediate vicinity. The Euclidian neighbourhood model was abandoned in favour of less computationally intensive models when research focus was shifted from biological modelling to mathematical optimization. Topological neighbourhoods unrelated to the locality of the particle came into use, including what has come to be recognized as a global neighbourhood, *gbest* model, where each particle is associated with and able to obtain information from every other particle in the swarm.

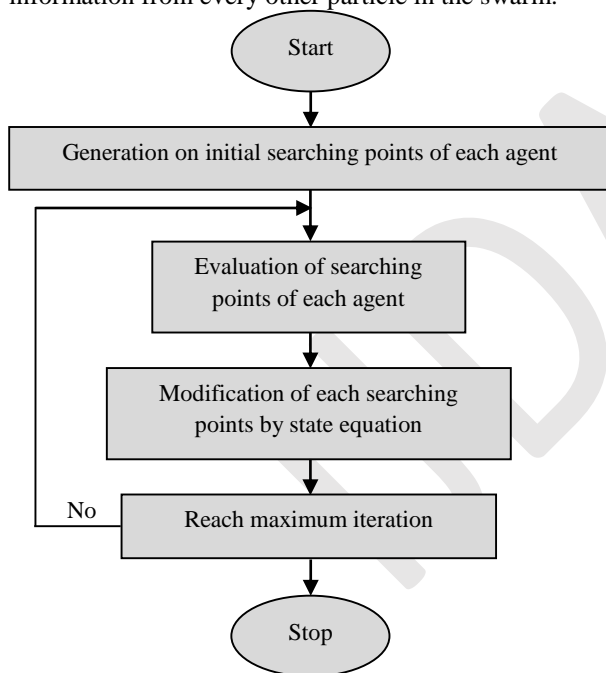


Figure 1: Flow chart of PSO

#### Particle Swarm Algorithm

1. Begin
2. Factor settings and swarm initialization
3. Evaluation
4.  $g = 1$
5. While (the stopping criterion is not met) do
6. for each particle
7. Update velocity
8. revise place and localized best place

9. Evaluation
10. End For
11. Update leader (global best particle)
12.  $g++$
13. End While
14. End

The PSO procedure has various phases consist of Initialization, Evaluation, Update Velocity and Update Position.

#### Initialization

The initialization phase is used to determine the position of the  $m$  particles. The random initialization is one of the most popular methods for this job. There is no assurance that a randomly created particle be a better answer and this will make the initialization more attractive.

A good initialization algorithm makes the optimization algorithm more efficient and reliable. For initialization, initial information or knowledge of the problem can help the algorithm to converge in less iterations.

#### Update Velocity and Position

In each iteration, each particle updates its velocity and position according to its heretofore best position, its current velocity and some information of its neighbours. Equation 8 is used for updating the velocity:

$$V_i^{(k+1)} = w * V_i^k + C_1 * rand_1 * (pbest_i^k - S_i^k) + C_2 * rand_2 * (gbest^k - S_i^k) \quad (8)$$

Where  $V_i^k$  is the velocity of  $i^{th}$  particle vector at  $k^{th}$  iteration;

$w$  is the weighting function;

$C_1$  and  $C_2$  are the positive weighting factors;

$rand_1$  and  $rand_2$  are the random numbers between 0 and 1;

$S_i^k$  is the current position of  $i^{th}$  particle vector  $h(n)$  at  $k^{th}$  iteration;

$pbest_i^k$  is the personal best of the  $i^{th}$  particle at the  $k^{th}$  iteration;

$gbest^k$  is the group best of the group at the  $k^{th}$  iteration.

The searching point in the solution space may be modified by the following equation:

$$S_i^{(k+1)} = S_i^k + V_i^{(k+1)} \quad (9)$$

The first term of equation (8) is the previous velocity of the particle vector. The second and third terms are used to change the velocity of the particle vector. Without the second and third terms, the particle vector will keep on “flying” in the same direction until it hits the boundary. Namely, it corresponds to a kind of inertia represented by the inertia constant,  $w$  and tries to explore new areas.

#### IV. IMPROVED PARTICLE SWARM OPTIMIZATION (IPSO)

The global search ability of conventional PSO is very much enhanced with the help of the following modifications. This modified PSO is termed as IPSO [10].

The two random parameters  $rand_1$  and  $rand_2$  of equation (8) are independent. If both are large, both the personal and social experiences are over used and the particle is driven too far away from the local optimum. If both are small, both the personal and social experiences are not used fully and the convergence speed of the technique is reduced. So, instead of taking independent  $rand_1$  and  $rand_2$ , one single random number  $r_1$  is chosen so that when  $r_1$  is large,  $(1 - r_1)$  is small and vice versa. Moreover, to control the balance of global and local searches, another random parameter  $r_2$  is introduced.

For birds collecting for food, there could be some rare cases that after the position of the particle is changed according to equation (8), a bird may not, due to inertia, fly toward a region at which it thinks is most promising for food. Instead, it may be leading toward a region which is in the opposite direction of what it should fly in order to reach the expected promising regions. So, in the step that follows, the direction of the bird's velocity should be reversed in order for it to fly back into promising region.  $sign(r_3)$  is introduced for this purpose. Both cognitive and social parts are modified accordingly. Other modifications are described below.

A new variation in the velocity expression (equation (8)) is made by splitting the cognitive component (second part of (8)) into two different components. The first component is called good experience component. That is, the particle has a memory about its previously visited best position. This component is exactly the same as the cognitive component of the conventional PSO. The second component is given the name bad experience component. The bad experience component helps the particle to remember its previously visited worst position. The inclusion of the worst experience component in the behavior of the particle gives additional exploration capacity to the swarm. By using the bad experience component, the bird (particle vector) can bypass its previous worst position and always try to occupy a better position. Finally, with all modifications, the modified velocity of the  $i^{th}$  particle vector at the  $(k + 1)^{th}$  iteration is expressed as in equation (10):

$$V_i^{(k+1)} = r_2 * sign(r_3) * V_i^k + (1 - r_2) * C_1 * r_1 * (pbest_i^k - S_i^k) + (1 - r_2) * C_2 * (1 - r_1) * (gbest^k - S_i^k) + (1 - r_2) * c_1 * r_1 * (S_i^k - pworst_i^k) \quad (10)$$

Where  $sign(r_3)$  is a function defined as:

$$sign(r_3) = \begin{cases} -1 & \text{Where } r_3 \leq 0.05 \\ 1 & \text{Where } r_3 > 0.05 \end{cases} \quad (11)$$

$V_i^k$  is the velocity of  $i^{th}$  particle vector at  $k^{th}$  iteration;  $r_1, r_2$  and  $r_3$  are the random numbers between 0 and 1;  $S_i^k$  is the current position of the  $i^{th}$  particle at the  $k^{th}$  iteration;

$pbest_i^k$  and  $pworst_i^k$  are the personal best and the personal worst of the  $i^{th}$  particle, respectively;

$gbest^k$  is the group best among all  $pbest$  for the group. The searching point in the solution space is modified by the equation (9) as usual.

#### V. SIMULATION AND RESULTS

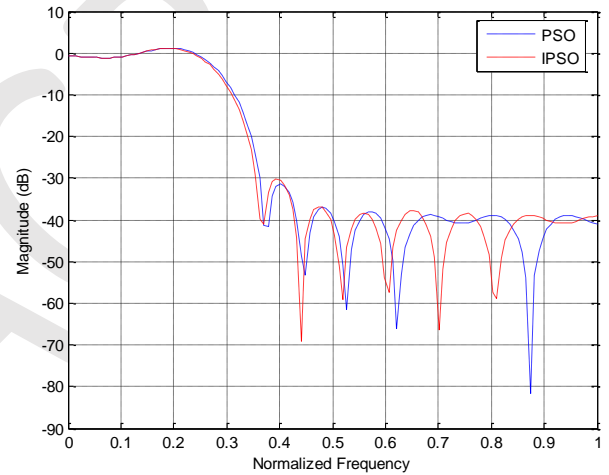


Figure 2: Magnitude (dB) plot of the low pass filter

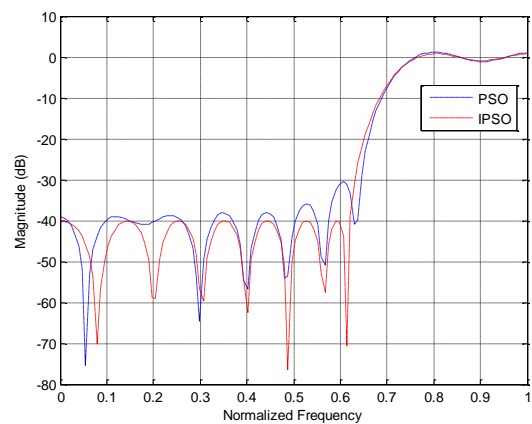


Figure 3: Magnitude (dB) plot of the high pass filter



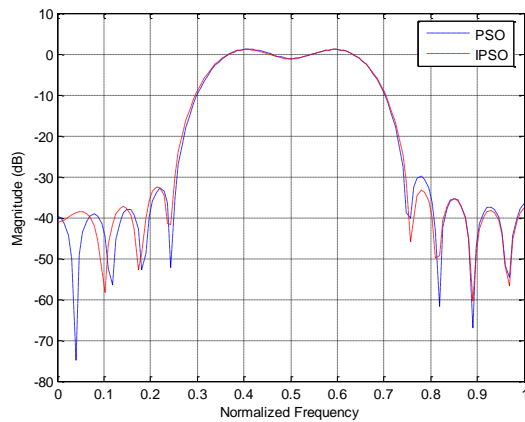


Figure 4: Magnitude (dB) plot of the band pass filter

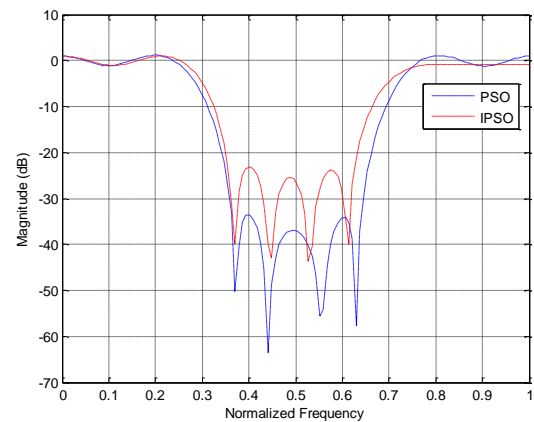


Figure 5: Magnitude (dB) plot of the band stop filter

Table 1: Comparative results

Method	Low Pass Filter		High Pass Filter		Band Pass Filter		Band Stop Filter	
	Ripple	Attenuation (dB)	Ripple	Attenuation (dB)	Ripple	Attenuation (dB)	Ripple	Attenuation (dB)
PSO	2.1467	4.1238	2.2851	114.93	2.2856	7.0939	2.2805	8.2834
IPSO	2.1829	4.1544	1.7386	114.45	2.2542	7.0283	1.0737	8.8318

## VI. CONCLUSION

This paper implemented improved particle swarm optimization (IPSO) for designing; low pass, high pass, band pass and band-stop filters. In this design, better results are obtained by varying the population size. This algorithm is better solution for designing of filters. For the sake of comparison conventional PSO is applied individually. Extensive simulation results justify that the proposed algorithm IPSO outperforms PSO in the accuracy of the magnitude response of the filter as well as in the convergence speed and is thus adequate for use in other related design problems.

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